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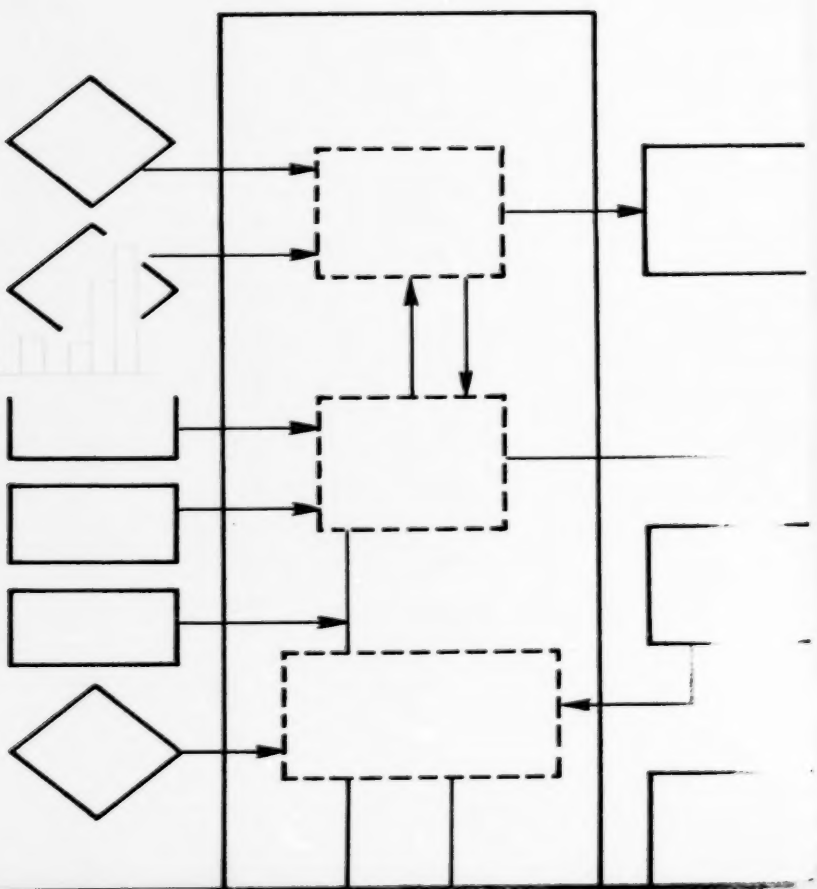

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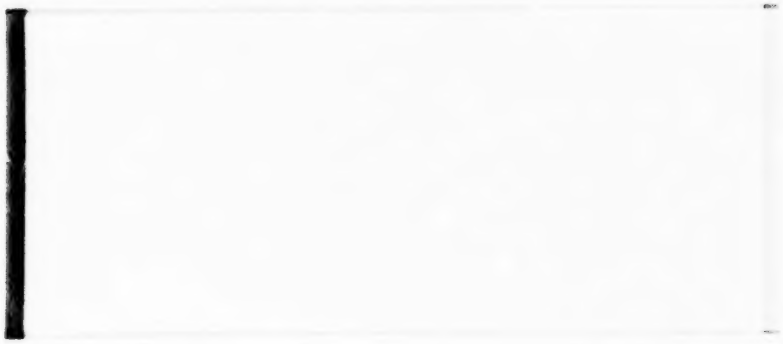

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ERRATUM

On page iv, the next to the last sentence should be read: "the elasticity of the fiscal multiplier "with respect" to the rate of subsidization may be "negative and" greater than one ..."

DISCUSSION PAPER NO. 238

Selective Economic Subsidization and Stabilization Policy in an Inflationary Environment: A Dynamic Aggregative Model

by François Delorme

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Résumé

Ce texte est consacré à l'étude de l'interdépendance entre une politique sélective de subvention et l'efficacité des politiques générales de stabilisation. Par politique sélective, on entend les prêts subventionnés qui sont destinés à une certaine catégorie d'investisseurs. Cette question semble avoir été laissée pour compte dans la littérature économique. Ceci s'explique par le fait que l'analyse de la subvention s'effectue surtout dans un cadre micro-économique tandis que celle des politiques de stabilisation relève de la macro-économie.

Le texte procède à l'analyse de cette interdépendance à partir d'un modèle IS-LM dynamique fermé, prenant en considération de façon explicite l'effet de la richesse, qui s'inscrit dans la lignée des modèles avec contrainte budgétaire du gouvernement élaborés par Ott et Ott (1965), Christ (1968) et Turnovsky (1975). Le modèle répartit les investisseurs en deux groupes, ceux qui bénéficient de prêts subventionnés et les autres qui opèrent sans subvention. La contrainte budgétaire du gouvernement et les attentes inflationnistes introduisent l'élément dynamique du modèle.

Le modèle est construit de telle sorte qu'il permette d'étudier comment la subvention modifie l'impact des politiques de stabilisation. Pour ce faire, la politique budgétaire en tant qu'agent de stabilisation et le prêt subventionné en tant qu'instrument sélectif de médiation sont explicitement intégrés au modèle en tant qu'instruments d'intervention.

On dérive ensuite les multiplicateurs d'impact et de long terme tout en procédant à une analyse de la stabilité du modèle à l'aide des conditions de Routh-Hurwitz. À partir des différents résultats théoriques, on procède à la simulation du modèle en vue de quantifier les différents multiplicateurs.

Les résultats montrent à partir d'hypothèses réalistes en ce qui a trait aux scénarios, que la subvention a généralement peu d'effets sur le multiplicateur fiscal. Cependant, sous certaines hypothèses plus radicales (mais non-extrêmes), l'élasticité du multiplicateur fiscal par rapport au taux de subvention peut être négative et supérieur à l'unité, ce qui signifie, en d'autres termes, qu'il tend à affaiblir l'impact de la politique fiscale.

Abstract

The paper examines the interdependence between a selective subsidization policy and the effectiveness of stabilization policies. A selective policy is one in which a subsidized loan is provided to a specific group of investors. This topic appears to have been neglected by the economic literature. One possible explanation is that the analysis of subsidization is based primarily on micro-economic factors, while that of stabilization policies rests on macro-economic aspects.

This interdependence is analysed through a closed dynamic IS-LM model with wealth, along the lines of the models developed by Ott and Ott (1965), Christ (1968) and Turnovsky (1975). The model divides investors into two groups: those receiving a subsidized loan, and those operating without a subsidy. The model's dynamics incorporates the government budget constraint and an adaptative process for inflationary expectations.

The purpose of the model is therefore to study how subsidization affects the impact of stabilization policies. To accomplish this, the model explicitly includes fiscal policy as a stabilizing agent, used as an instrument of intervention, and the subsidized loan, used as a selective instrument of intermediation. The impact and long-term multipliers are then derived while analysing the model's stability using the Routh-Hurwitz conditons. The

various theoretical results are used to simulate the model, in order to quantify the various multipliers.

The results demonstrate that under realistic assumptions, subsidization generally has little effect on the fiscal multiplier. Under some more radical (but not extreme) assumptions, however, the elasticity of the fiscal multiplier to the rate of subsidization may be greater than one, indicating that it weakens the impact of fiscal policy.

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1. Introduction

This discussion paper was prepared as a background research for the Economic Council of Canada's report on government credit, Intervention and Efficiency, (E.C.C., 1982). The primary objective of this paper is to analyse the effect of subsidized loans granted by government through its agencies or departments on the effectiveness of stabilization policies. Aside from the fact that the government wishes to eliminate major swings in business cycle, it claims to pursue certain specific objectives, such as increasing the economy's potential. By using fiscal and monetary policies to accomplish this, it must ensure that the objectives set have every chance of being achieved. Even thougy it is not designed only in such a perspective, the subsidized loan is part of the dynamics of stabilization policy. A subsidy accompanies a loan issued by the government to a producer. In so doing, government favours a certain sector of the economy by subsidizing the costs of the financial funds that this sector requires. The higher the proportion of the subsidy, the more the advantages that accrues to the sector.

The subsidized loan as an instrument must be viewed in the more general perspective of economic stabilization. In this context, one serious question arises: Does subsidization alter the effectiveness of stabilization policies?

For example, let us assume that government introduces a series of policies with a firmly restrictive effect on the economy, such as a generalized cut in public expenditures on goods and services,

an increase in personal income tax rates or an increase in interest rates. Economic theory predicts that with no compensating action, these measures will slow economic growth. If, on the other hand, government wishes to protect some sectors of the economy from the burden of its restrictive policies, it can use subsdized loans as a selective financial instrument. In doing so, however, government may weaken the impact of its stabilization policies because, in most cases, the subsidy will have an expansionary effect on the economy. Government would then be forced to take additional steps in order to ensure that its restrictive policies will achieve their goals. The result would be an overly difficult situation for individuals, as well as for the sectors not receiving government assistance.

To analyse this issue, we have developed a theoretical and analytical model representing the economy as a whole and explicitly including public expenditures on goods and services as an agent of stabilization, as well as the subsidized loan as an instrument of intermediation. As we shall see later, the link between these two elements is government budget constraint - in other words, the financing of the government deficit. With this model, we plan to study the effects of fiscal policy (public expenditures) on the economy and, specifically, on its interdependence with subsidization.

The paper first describes the model used, as well as the underlying constraints for its optimum control. In a second part, we simulate the model, using alternative scenarios. Finally,

we attempt, in our concluding remarks, to place the model in a realistic perspective.

2. The Model

The interdependence between subsidization and stabilization has not generated much research by economists, although each element, taken separately, has been studied extensively.[1] One possible explanation may reside in the discretionary character of the subsidy and the fact that the analysis of this subsidization is based primarily on micro-economic considerations while that of stabilization policies is based on macro-economics.

The model that we have constructed is based on macro-economic models developed to study the role and impact of stabilization policies. Interest is growing in models incorporating government budget constraint. Initially studied by Ott and Ott (1965) and by Christ (1967, 1968), the role of the government budget constraint has proven to be an important dynamic element, which reality imposes on macro-economic models in the medium- and long-run.

The analysis is based on a simple extension of the closed dynamic IS-LM model, expressed in continuous time and based on several restrictive hypotheses. The model's dynamics is described by a process of accumulation of wealth and the evolution of

inflationary expectations. The model is comparable to those developed by Christ (1978), Turnovsky (1979), and Mitchell (1981), which were aimed at studying the impact of certain stabilization policies as well as the stability conditions of underlying models in an inflationary environment.

The real sector of our economy is described by the following equations:

(1a) $C = C(Yd)$

(1b) $Yd = Y - T + rb - \pi W$

(1c) $T = u(Y + rb)$

(1d) $W = b + m$

(2) $I_1 = F_1(r-\pi) = \gamma_0 + \gamma_1 (r - \pi) \qquad \gamma_0>0,\ \gamma_1<0$

(3) $I_2 = F_2 \ (r - \pi)(1 - \phi) =$
$\theta_0 + \theta_1(r - \pi)(1 - \phi) \qquad \theta_0>0,\ \theta_1<0$

(4) $Y = c\{(1 - u) \ Y + r \ (W - m) \ - \pi W\} +$
$\gamma_0 + \gamma_1 \ (r - \pi) + \theta_0 + \theta_1 \ (r - \pi) \ (1 - \phi)$
$+ \phi W + G$

where Y = real income

Yd = real disposable income

C = real consumption

T = real taxes (assumed to be proportional)

r = nominal interest rate

b = stock of real government bonds

π = expected inflation rate

W = real private wealth

u = income tax rate

I_1 = unsubsidized real investment

I_2 = subsidized real investment

ϕ = rate of subsidy

m = real money supply

G = real government expenditures

c = marginal propensity to consume

and where γ_0, γ_1, θ_0, θ_1, ϕ are parameters, and ϕ is positive.

Equation (4) represents the condition of equilibrium, in the product market that is partially derived from (1), (2) and (3), where aggregate private demand increases in conjunction with real net income and real wealth, and decreases when real interest rates rise.[2] Equations (1a) to (1d) indicate that real consumption depends on real disposable income, defined as real net income plus interest accrued on government bonds, less expected capital losses on wealth due to the inflation tax.[3]

Equation (2) implies that unsubsidized investment depends on the interest rate, while subsidized investment depends on the

subsidized interest rate as specified by equation (3). The subsidization instrument will be activated through this last function. The reader should note that this concept of investment corresponds to a narrow definition. In fact, variables other than the interest rate might have been included as determinants of investment decisions.

Some conditions must be imposed here on the overall behaviour of the IS curve. From equation (4), we see that an increase in the real interest rate will have two opposite effects: a positive income effect (via (1b)) and a negative substitution effect (via (2) and (3)). If we assume that the substitution effect dominates, we must place the following constraints on the model:

$$(5a) \qquad \partial y/\partial r = c(1-u)(W-m) + \gamma_1 + \theta_1 (1-\phi) \,/\, 1-c(1-u) \;<\; 0$$

$$(5b) \qquad \partial y/\partial \pi = -\; cW + \gamma_1 + \theta_1 (1-\phi) \,/\, 1-c(1-u) \;>\; 0$$

Similarly, an increase in W will generate an income effect and a wealth effect. The sign of the income effect depends on the form the additional wealth takes. If the wealth is channelled through real bonds, this means that the monetary policy is accommodating,[4] i.e., that the nominal money supply is adjusting to meet the needs of transactions, which grow at the rate of inflation.[5] Normally, a pure fiscal policy is associated with a fixed money supply in real terms. This means that the deficit is pure bond financing. However, if the policy is specified in real terms, this does not

contradict the fact that the various instruments available to monetary authorities have nominal variations. If in addition, we assume a positive real interest rate, the income effect will be

$$c\ (1 - u)\ r - \pi\ > 0$$

and the total effect of an increase in W will be

$$(5c) \qquad \partial Y/\partial W = c\ (1-u)\ r-\pi\ +\phi\ /\ 1-c(1-u)\ > 0$$

If it is the bonds that represent the accommodating policy, in that the authorities are attempting to keep the real value of government bonds unchanged, the income effect will be $-c\pi$ (the wealth-generating channel is now money, which provides no nominal interest), and (5c) will become negative. In this case, the deficit is pure money financing.

If we assume that the positive effect dominates in the latter case, we must impose the following restriction on the model:

$$(5d) \qquad \partial Y/\partial W = \phi - c\pi\ /\ 1 - c(1 - u) > 0$$

The demand for real money balances is represented by the equation

$$(6) \qquad \frac{M}{P} = m = L(Y,\ r,\ W,) = \alpha Y + \beta r + \rho W$$

where M = nominal money supply

P = level of prices

and α, β, ρ are parameters and α, $\rho > 0$, $\beta < 0$.

Equation (6) therefore gives us the equilibrium in the money market.[6]

In a static context, we have three endogenous variables, Y, r and P, with $\bar{m}$[7] and two equations. The closing of the model was covered in a seminal article written by Friedman (1970). The model can be closed in Keynesian fashion where the nominal wage is exogenous and it is assumed that firms will bring prices into line with the marginal costs. We then consider price as the exogenous variable to solve (4) and (6) for Y and r. To close the system *à la* Friedman, Y is assumed exogenous and made equal to a given potential Y, leaving P and r endogenous.

Taking these two methods as the extremes, we will adopt an intermediate closing method by using a conventional price adjustment equation. Thus Y, r and P are determined endogenously. The Phillips curve with an "expectative" form is extensively used in this respect in theoretical models.[8] Initially developed by Friedman (1968) and Phelps (1968), it is expressed as follows:

$$(7) \qquad \frac{\dot{S}}{S} = s = \chi_1 U + b_1 \pi$$

$$\chi_1 < 0, \; 0 \leq b_1 \leq 1$$

where s = the growth rate of wages

U = labour tightness variable (unemployment rate)

and χ_1 and b_1 are parameters.

The coefficient χ_1 represents the short-term relationship between the tightness in the labour market (measured by U) and the rate of wage inflation for a given expected rate of inflation. In turn, the coefficient b_1 demonstrates to what extent an increase in inflationary expectations affects the growth rate of wages.

By excluding the capital stock,[9] we obtain the production function

$$(8) \qquad Y = f(N) \qquad f_N > 0$$

where N = employment.

If U is sufficiently small for f(N) to be represented by a linear approximation around the full employment equilibrium point, then[10]

$$(9) \qquad U = \frac{N^* - N}{N^*} = \omega(Y - Y^*), \quad \omega < 0$$

with the asterisk designating the potential value. Prices are at a constant mark-up over unit labour costs so that, in the absence of technological growth, productivity remains constant and

$$(10) \qquad p = s$$

where p = the growth rate of prices.

By eliminating s and U from (7) through (9) and (10), we obtain the price adjustment equation:

$$(11) \qquad \frac{\dot{P}}{P} = p = a_1 (Y - Y^*) + b_1 \pi$$

$$a_1 > 0, \qquad 0 \leq b_1 \leq 1, \text{ where, } a_1 = \omega \chi_1$$

Equation (11) describes how p varies in reaction to an imbalance in the goods market and to inflationary expectations. Although the theoretical basis for the Phillips curve is still debated, equation (11) will be adequate for our needs.[11]

Equations (1b), (1d), (4), (6) and (11) describe a system of instantaneous relationships which determine Y, Yd, r, p and b (or m) with the assistance of the predetermined variables W, π, m (or b) and other exogenous variables.[12]

The dynamics of the model are described by the following equations:

$$(12) \qquad \dot{W} = G - uY + (1 - u)\, r\, (W - m) - pW +$$

$$r\phi\, \theta_0 + \theta_1 (r - \pi)(1 - \phi)$$

$$(13) \qquad \dot{\pi} = \delta(p - \pi)$$

$$\delta > 0$$

In equation (12),[13] we see government budget constraint expressed in real terms.[14] The term r(W - m) represents interest payments on the public debt, and the term -pW equals the inflation tax on the public debt.[15] The last term is the value of the subsidy. Equation (12) also represents the process of wealth accumulation in the private sector by using the identity (1d). Equation (13) describes an adaptative process of the change in inflationary expectations.

In order to simplify our model[16] we have excluded the capital accumulation process which several authors have attempted to include in their respective macro-dynamic models.[17]

We will analyse the effect of m, G and ϕ in the following cases:

a) fixed money supply in real terms (the deficit is financed by bond issues exclusively) ($m = \bar{m}$);

b) fixed stock of government bonds in real terms (the deficit is financed by money creation exclusively) ($b = \bar{b}$).

In this latter case, the system of equations (4), (6), (11)-(13) is expressed as follows:

$$(14'a) \quad O = c\{(1-u)(Y+r\bar{b}) - \pi W\} + \gamma_0 + \gamma_1(r-\pi) + \theta_0 + \theta_1(r-\pi)(1-\phi) + \phi W + G - Y$$

$$(14'b) \quad O = \alpha Y + \beta r + \rho W - (W - \bar{b})$$

$$(14'c) \quad O = a_1 (Y - Y^*) + b_1 \pi - p$$

$$(15'a) \quad \dot{W} = G - uY + (1-u)\, r\bar{b} - pW + r\phi\, \theta_0 + \theta_1 (r-\pi)(1-\phi)$$

$$(15'b) \quad \dot{\pi} = \delta(p-\pi)$$

We will analyse the latter case only in the empirical part. The comparison of financing methods for fiscal and monetary policies can then be made in the context of the dynamic adjustment to the disturbances introduced into the model.

For the initial development of the model (financing through the issuance of bonds), we will conduct the analysis in the following way: first, we will examine the short-term impacts; then the stability of the system; and, finally, the long-term effects.

3. Theoretical Results

From equations (4), (6), (11), (12) and (13), we can express a linearized dynamic model:

$$(14a) \quad O = c\{(1-u)\, Y + r(W-\bar{m}) - \pi W\} + \gamma_0 + \gamma_1 (r-\pi) + \theta_0 + \theta_1 (r-\pi)(1-\phi) + \phi W + G - Y$$

(14b) $0 = \alpha Y + \beta r + \rho W - \bar{m}$

(14c) $0 = a_1 (Y - Y^*) + b_1 \pi - p$

(15a) $\dot{W} = G - uY + (1-u)\, r(W-\bar{m}) - pW + r\phi \quad \theta_0 + \theta_1 (r-\pi)(1-\phi)$

(15b) $\dot{\pi} = \delta(p - \pi)$

The three equations (14a, b, c) determine the instantaneous equilibrium for Y, r and p in terms of G, $\bar{m}$, π, W and ϕ, while the equations (15a, b) describe the behaviour of the model (with (14)) in a state of disequilibrium. The impact multipliers are given in Table 1.

a) Short-term Effects

Short-term results show that an increase in public expenditures on goods and services that is financed through the issuance of bonds will increase the level of income, the rate of interest, and the rate of inflation over the short term. An expansionary monetary policy will reduce the nominal interest rate, which will induce indetermination with respect to income and the rate of inflation. The positive effect generated by the increase in m is partially offset by a reduction in interest payments attributable to the decline in r. We can, however, assume that the stimulating effect will prevail. An increase in the rate of subsidization has

Table 1

Impact Multipliers for Selected Exogenous Variables

On	Effect of G	$\bar{m}$	ϕ	π
Y	$\frac{-\beta}{\sigma} > 0$	$\frac{(1-u)\,c r\beta - [\gamma_1 + \theta_1(1-\phi)] + (1-u)\,c(W-\bar{m})}{\sigma} \lesseqgtr 0$	$\frac{\beta\theta_1(r-\pi)}{\sigma} > 0$	$\frac{\beta[c W + \gamma_1 + \theta_1(1-\phi)}{\sigma} > 0$
r	$\frac{\alpha}{\sigma} > 0$	$\frac{-(1-c\,(1-u)) - \alpha c r(1-u)}{\sigma} < 0$	$\frac{-\alpha\theta_1(r-\pi)}{\sigma} > 0$	$\frac{-\alpha[c W + \gamma_1 + \theta_1(1-\phi)]}{\sigma} > 0$
p	$\frac{-a\beta}{\sigma} > 0$	$\frac{-a_1[[\gamma_1 + \theta_1(1-\phi) + c\,(1-u)(W-\bar{m})] - c\,(1-u)r\beta]}{\sigma} \lesseqgtr 0$	$\frac{a[\beta\theta_1(r-\pi)]}{\sigma} > 0$	$1 + \frac{a\beta(c W + \gamma_1 + \theta_1(1-\phi))}{b\sigma} > 0$

$$\sigma = -\left[\beta(1-c(1-u)) + \alpha[\gamma_1 + \theta_1(1-\phi) + c\,(1-u)(W-\bar{m})]\right] > 0$$

a positive impact on all the endogenous variables. Finally, an instantaneous increase in π leads to an increase in income, and its total effect on the rate of inflation will exceed the partial response determined from the expectations parameter b_1 of the Phillips curve, which implies a drop in the real interest rate. Let us analyse these effects more thoroughly.

According to the model, an expansionary fiscal policy would have a positive effect on output (and employment), but it would also lead to a higher rate of inflation. Furthermore, the more insensitive investors are to variations in interest rates and the more the demand for money is elastic in relation to these variations, the greater the impact of the multiplier. For prices, the higher the multiplier and the more strongly people react to the imbalances in the goods market, the greater the impact of fiscal expansion on the rate of inflation. It must be noted that part of the expansionary impact of fiscal policy results from the fact that the model assumes a proportional rate of taxation. With a progressive tax structure, the multiplier will be smaller and may even be deflationary.

An increase in government expenditures leads to an increase in the nominal interest rate. The increase in G stimulates output; the transactions demand for money is therefore greater and, in the context of a non-accommodating monetary policy in real terms ($m = \bar{m}$), this increase in G exerts upward pressure on interest rates.

An increase in $\bar{m}$ will have an indeterminate effect on Y. The expansionary effect of the increase in the monetary base is offset by the term $(1-u)cr\beta$, which represents the downward effect of the decrease in interest payments that is caused by the drop in interest rates. However, empirical studies have generally demonstrated that the positive effect predominates. If that is so, monetary policy will also be inflationary in the short run.[18]

An increase in the rate of subsidization has a positive effect on income, the level of the nominal interest rate, and the inflation rate. However, in addition to finding that the greater the interest elasticity of the demand for money, the greater the multiplier effect of the subsidy, we also discover that the increased elasticity of investment in relation to the subsidized interest rate triggers a positive impact on income. In effect, since an increase in the rate of subsidization is seen as a decrease in the actual interest rate paid, an increase in investment and output results. With respect to the effect of the endogenous variables, care must be taken not to confuse effect and consequence. An increase in ϕ is equal to a decrease in $(r-\pi)$ in equation (3), which stimulates the subsidized investment and, therefore, Y. This does not, however, mean that the impact of an increase in ϕ is to decrease r since, as we see in Table 1, $\partial r/\partial \phi > 0$.

The impact on the endogenous variables is greater under a selective subsidization policy than under an expansionary fiscal

policy. As long as θ_1 and $(r-\pi)$ do not equal 0, an increase in ϕ has a greater impact on:

1) output, because the decrease in $(r-\pi)$ stimulates subsidized investment;

2) the interest rate, because with a greater Y, the effect of the transactional demand will be greater and thus so will the upward pressure on r;

3) the rate of inflation, because the additional output created by ϕ adds to the excess demand pressure on the goods market, resulting in a higher p (it also has effects on the supply side but with lags).

An increase in the anticipated rate of inflation is channeled through two investment functions as well as through the price adjustment equation.[19]

Finally, the last column in Table 1 shows that an increase in anticipations exerts a positive effect on the real sector of the economy and on the nominal interest rate, as well as on the current rate of inflation.[20]

b) Stability Analysis Using the Routh-Hurwitz Conditions

An examination of the stability conditions for the bond-financing case appears to indicate that the system is potentially unstable.[21] A thorough analysis reveals that:

1) a positive wealth effect in the IS curve is a necessary element for the model's stability;

2) the existence of a wealth effect in the LM curve has a destabilizing influence.[22]

The economic intuition concealed in the Routh-Hurwitz conditions[23] deserves special attention. Assume that the economy, on its long-term growth path, i.e., $\dot{\pi} = \dot{W} = 0$, is "disturbed" by an exogenous shock, such as an increase in the rate of subsidy ϕ.

The immediate effect of this initiative will be to increase Y, p and r[24] and to create a deficit through the term $r\phi[\theta_1(r-\pi)(1-\phi)]$ in the budget constraint. Deficit financing will create an accumulation of wealth, while the rise in p will result in the economic agents revising their inflationary expectations upwards. These increases, combined with W and π, will have subsequent effects in the context of the model's operation. Because $\partial p/\partial \pi > b_1$, π will tend to grow again, introducing a destabilizing agent into the model. In addition, the simultaneous increases of Y and

π will have a positive impact on r, thus adding to the public debt through increased interest payments and subsidy expenses.

It appears that if the wealth effect on aggregate demand is non-existent, an increase in W will reduce p, which reduces the inflation tax on wealth - a stabilizing factor for the model's dynamics.[25]

If this were so, the Routh-Hurwitz conditions would not be met and the model would undoubtedly be unstable. On the other hand, if the wealth effect is positive, it is then possible that the increase in p created by the increase in W will offset other destabilizing elements related to the model's dynamics.

Similarly, if the wealth effect on the demand for money is large, it is then possible that $\partial p/\partial W$ will be deflationary and destabilizing. This is why a positive wealth effect on aggregate demand is a necessary condition for the model's stability, while $\rho > 0$ exerts a destabilizing influence.

We note that the predominant source of instability in the model arises from the interest payments on the public debt and subsidy disbursements, which require continuous financing. This phenomenon is, however, offset by fiscal receipts and the inflation tax.[26]

c) Long-term Effects

Stationary equilibrium therefore requires two conditions drawn from the dynamic equations (15a) and (15b).

$$(16a) \quad G - uY + (1-u)\, r(W-\bar{m}) + r\phi\ \theta_0 + \theta_1 (r-\pi)(1-\phi) = pW$$

$$(16b) \quad p = \pi$$

Thus, the expected inflation rate must equal the current inflation rate; in other words, expectations must be perfectly realized and the inflation tax must offset the deficit, interest payments and subsidization expenditures. By substituting the conditions of (16) in system (14), we obtain the equations for the model in the steady state.

$$(17a) \quad Y - c\,\{Y - G - r\phi\ \theta_0 + \theta_1 (r-p)(1-\phi\,\} - \theta_0 - \theta_1 (r-p)(1-\phi) - \gamma_0 - \gamma_1 (r-p) - \phi W - G = 0$$

$$(17b) \quad \bar{m} - \alpha Y - \beta r - \rho W = 0$$

$$(17c) \quad (1 - b_1)\, p - a1\, (Y - Y^*) = 0$$

$$(17d) \quad G - uY + r(W-\bar{m})(1-u) - pW + r\phi\ \theta_0 + \theta_1 (r-p)(1-\phi) = 0$$

$$(17e) \quad \pi = p$$

Table 2

Long-Term Multipliers of Selected Exogenous Variables

Over	Effect of G	ϕ	$\bar{a}$
Y	$(b-1)\left[I(HB-CK)+KA-BF\right]/\Sigma$	$(b-1)\left[J(HB-CK)+M(KA-BF)\right]/\Sigma$	$(b-1)\left[L(KA-BF)+HA-CF\right]/\Sigma$
r	$\left(I\left[K(Ea+u(1-b)-H\alpha(1-b)\right]+\alpha F(1-b)-K\left[(1-c)(1-b)+aD\right]\right)/\Sigma$	$\left[J\left[K(EA+u(1-b)-H\alpha(1-b)\right]-M\left[DKa+(1-b)(K(1-c)-\alpha F)\right]\right]/\Sigma$	$\left[-a\left[D(H-KL)-EF\right]-(1-b)\left[(1-c)(H-KL)+F(\alpha L-u)\right]\right]/\Sigma$
p	$a\left[K(CI-A)+BF\right]/\Sigma$	$-a\left[B\left[HJ-MF\right]+K(MA-CJ)\right]/\Sigma$	$a\left[A(H-KL)+I(BL-C)\right]/\Sigma$

where $\Sigma=-a\left[B(HD-EF)+K(AE-CD)\right]-(1-b)\left[(1-c)(HB-CK)-\alpha(HA-CF)+u(KA-BF)\right]>0$ required for local stability.

$A=-\left\{Y_1-c\left[r\phi\theta_1(1-\phi)-\phi\left[\theta_0+\theta_1(r-p)(1-\phi)\right]\right]\right\}$

$B=\beta<0.$

$C=-\left\{(W-\bar{m})(1-u)+r\phi\theta_1(1-\phi)+\phi\left[\theta_0+\theta_1(r-p)(1-\phi)\right]\right\}$

$D=Y_1-cr\phi\theta_1(1-\phi)+\theta_1(1-\phi)$

$E=W+r\phi\theta_1(1-\phi)$

$F=-\rho<0$

$H=-\left[r(1-u)-p\right]$

$I=(1-c)>0$

$J=c\left[r\phi\theta_1(r-p)-r(\theta_0+\theta_1(r-p)(1-\phi))\right]-\theta_1(r-p)$

$K=\rho>0$

$L=-r(1-u)<0$

$M=-\left[r\phi\theta_1(r-p)-r\left[\theta_0+\theta_1(r-p)(1-\phi)\right]\right]$

If we wish to express the "accelerationist" tendency of the transmission of inflation, we then assume $b_1 = 1$; therefore, y becomes exogenous and equals its potential value, since it is no longer affected by discretionary policies.

From the equations (17a - 17e), we can derive the long-term multipliers presented in Table 2.

As the system has become quite complex, neither a positive or negative value can be assigned in theory to each of the derivatives with any uncertainty. However, we will attempt to determine the sign of those relevant to our analysis in the empirical part.

4. Empirical Results

To quantify some of the impacts discussed in the theoretical section and to specifically study the interdependent relationship between subsidization and fiscal policy, we simulated the model, starting with a hypothetical economy reduced to scale, in which the units of measure and of time were selected arbitrarily.

To generate our data bank, we assigned a priori values to the model's parameters and variables, and solved the system for r, b and m. The system of non-linear equations (17a), (17b) and (17d) can be studies as a determinant or r, b and m for given values of Y, p, G, θ_0, θ_1, γ_0, γ_1, ψ, α, β, ρ, and ϕ, and of the remaining

variables and parameters. If a solution can be found, the values of r, b and m, in conjunction with those of the other variables, will constitute a stationary solution to the model. Equation (17c) will provide the corresponding Y, while (17e) indicates that in the steady state, the present and expected inflation rates are identical.

Our sample, which includes only the non-negative solutions, represents 20.41 per cent of the possible solutions (35,264/172,800). Of this sample, only 27.16 per cent (9,577/35,264) meet both the conditions of stability and the restrictions in (5). This new subset is the starting point for our empirical analysis.

The data used in our simulations therefore meet certain conditions:

1) the conditions of long-term equilibrium in (17) for selected parameters;

2) the stability conditions specified in Appendix 1. The system's adjustment to the Routh-Hurwitz conditions guarantees adjustment paths with oscillations of decreasing magnitude. The study of these paths is especially critical because financing through bond issues is notoriously less stable than financing through the printing of money.

We selected 25 scenarios from the stable sample for more thorough analysis, choosing those with the most realistic assumptions and results (Table 3).[27] We note that the b/W ratio is fairly high because of the government's budget constraint. This quotient is not excessive, however, if we consider that m constitutes the monetary base and that physical capital is not included in the model. Bonds therefore take the place of all interest-bearing assets. Before proceeding with the actual empirical analysis, we should examine the model's stability when it is subjected to various hypotheses related to bond financing.

a) The Model's Stability: Study of Time Paths

To study the model's stability, the system's results following equation (17) have been used, as well as those resulting from the stability conditions. This gives us an idea of the model's stability for different values of the key parameters. This exercise is illustrated in Table 4. It will be noted that the degree of stability shows almost no variation with subsidy coverage. Whether ϕ is small or large, the number of stable solutions does not significantly exceed one-fifth of the total. In the case of a_1, the level of stability again remains the same whether the reaction of prices to excess demand in the consumer goods market is normal or strong. The results for b_1 reveal that about 15.8 per cent of the cases are stable. Nguyen and Turnovsky (1979), however, demonstrated that a value of one for b_1 is very unstable in an accommodating monetary policy. The

ble 3

set of Stable Scenarios

narios	ϕ	ψ	α	β	ρ	γ_1	θ_1	a_1	b_1	δ	r	b	m	w	y	p
1	0.1	0.08	0.05	-100	0.2	-603	-111.1	0.4	0.4	0.4	0.18	427.0	90.3	517.3	100	0.12
2	0.1	0.08	0.05	-100	0.2	-603	-55.5	0.4	0.4	0.4	0.18	403.4	84.4	487.8	100	0.12
3	0.1	0.08	0.05	-100	0.2	-603	-11.1	0.4	0.4	0.4	0.18	384.7	79.8	464.5	100	0.12
4	0.1	0.08	0.10	-250	0.4	-603	-111.1	0.4	0.4	0.4	0.23	616.7	331.5	948.2	100	0.12
5	0.1	0.08	0.10	-250	0.4	-603	-111.1	0.4	0.4	0.4	0.23	575.6	304.8	880.4	100	0.12
6	0.1	0.08	0.10	-250	0.4	-603	-11.1	0.4	0.4	0.4	0.23	543.0	283.5	826.5	100	0.12
7	0.9	0.08	0.05	-500	0.4	-603	-999.9	0.4	0.4	0.4	0.20	531.8	191.3	723.1	100	0.12
8	0.9	0.08	0.05	-500	0.4	-603	-500.0	0.4	0.4	0.4	0.20	484.2	163.5	647.7	100	0.12
9	0.9	0.08	0.05	-500	0.4	-603	-99.9	0.4	0.4	0.4	0.20	446.4	141.8	588.2	100	0.12
10	0.9	0.08	0.05	-500	0.4	-603	-999.9	0.4	0.4	0.4	0.23	619.5	324.8	944.3	100	0.12
11	0.9	0.08	0.05	-500	0.4	-603	-500.0	0.4	0.4	0.4	0.23	572.5	294.8	867.3	100	0.12
12	0.9	0.08	0.05	-500	0.4	-603	-99.9	0.4	0.4	0.4	0.22	535.3	271.3	806.6	100	0.12
13	0.1	0.1	0.30	-10	0.1	-603	-111.1	0.4	0.4	0.4	0.18	337.3	68.8	406.1	100	0.12
14	0.1	0.1	0.30	-10	0.1	-603	-55.5	0.4	0.4	0.4	0.18	322.2	67.1	389.3	100	0.12
15	0.1	0.1	0.30	-10	0.1	-603	-11.1	0.4	0.4	0.4	0.18	310.0	65.7	375.7	100	0.12
16	0.1	0.1	0.30	-10	0.4	-603	-111.1	0.4	0.4	0.4	0.26	561.6	420.0	981.6	100	0.12
17	0.1	0.1	0.20	-10	0.4	-603	-55.5	0.4	0.4	0.4	0.26	519.0	375.0	894.0	100	0.12
18	0.1	0.1	0.10	-10	0.4	-603	-11.1	0.4	0.4	0.4	0.25	483.2	334.6	817.8	100	0.12
19	0.1	0.1	0.20	-250	0.01	-603	-11.1	0.4	0.8	0.9	0.18	316.2	123.8	440.0	100	0.12
20	0.9	0.1	0.30	-10	0.1	-603	-999.9	0.4	0.4	0.4	0.18	413.0	77.2	490.2	100	0.12
21	0.9	0.1	0.30	-10	0.1	-603	-500.0	0.4	0.4	0.4	0.18	322.4	67.1	389.5	100	0.12
22	0.9	0.1	0.10	-10	0.2	-603	-99.9	0.4	0.4	0.4	0.19	339.4	94.9	434.3	100	0.12
23	0.9	0.1	0.30	-250	0.4	-603	-999.9	0.4	0.4	0.4	0.24	516.1	294.2	810.3	100	0.12
24	0.9	0.1	0.30	-250	0.4	-603	-500.0	0.4	0.4	0.4	0.24	476.3	269.0	745.3	100	0.12
25	0.9	0.1	0.30	-100	0.2	-603	-99.9	0.4	0.4	0.4	0.19	343.6	99.3	442.9	100	0.12

Table 4

Stability of the Model for Various Values of Some Parameters (per cent)

	ϕ		ϕ				
	0.08	0.1	0.1	0.3	0.5	0.7	0.9
%	13.01	14.14	19.97	20.04	20.03	19.95	20.01

	a_1		b_1		δ		ρ	
	0.4	0.8	0.4	0.8	0.4	0.9	[0.1-0.4]	0.4
%	15.77	16.08	15.80	15.76	15.45	15.11	14.35	12.81

results for ρ are surprising compared to those obtained in the study mentioned above. ρ has a destabilizing effect on the model and should therefore trigger increasing instability as its value grows. A small value of ρ, however, results in only 14.35 per cent of the cases being stable, while a value of 0.4 makes no fewer than 12.81 per cent of the cases stable. One possible explanation may be that our model is stable by construction, and therefore is affected very little by ρ.[28] A large value of ρ does not, however, rule out the possible occurence of some ambiguous situations. When $\rho = 0.4$, 12.11 per cent of the long-term budget multipliers register a negative yet stable value. No adequate explanation for this has yet been found.[29] Table 4 also shows that as ψ increases, the model's stability tends to grow. Finally, none of these results appear to be significantly affected by the value of ϕ.

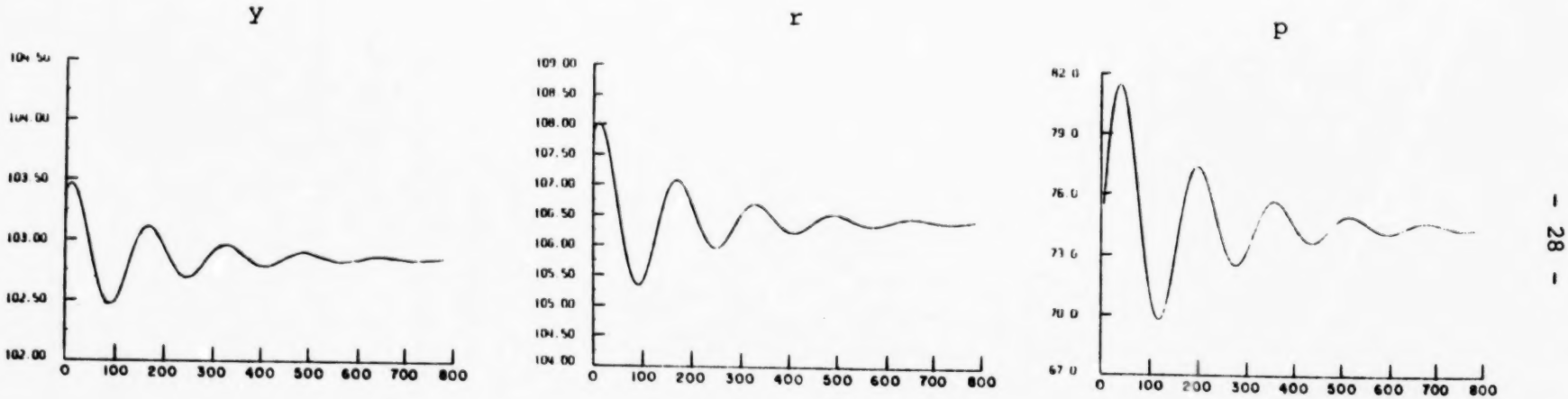
b) Dynamic Analysis of Impacts

The dynamic adjustment path of the system under expansionary fiscal and monetary policies is an interesting subject for study. An analysis of the behaviour of these policies in terms of the various forms of government financial policies would seem relevent.

Let us first study the model's behaviour after a 5 per cent shock in government expenditures. We will find that the empirical results generally confirm those derived in the theoretical

Chart 1

Dynamic Adjustment of an Expansionary Fiscal Policy Under a $m=\bar{m}$ Policy, (Scenario 19)[1]



1 C.f. note 30, $y_0=100.0$, $r_0=100.0$ and $p_0=66.66$.

section. Thus, an expansionist fiscal policy has an instant positive effect on Y, r and p. Even more important, however, the model's dynamics as specified creates cycles in the adjustment until the steady state is attained.

These oscillations are particularly pronounced when the fiscal policy is under a $m=\bar{m}$ policy (Chart 1).[30] The shock of G, corresponding to a short-term multiplier of about 0.7, helps to increase W, but the increases in Y, r and p and the decrease in the real interest rate will tend to offset this effect. To fully understand the dynamics imposed by budget constraint (15a) on the model, (1a) can be rewritten:

$$(18) \qquad \dot{W} = \dot{b} = G-uY + (r-p)b - p\bar{m} - urb + r\phi[\theta_0+\theta_1(r-p)(1-\phi)]$$

We see from equation (18) that an increase in G produces an increase in W and also triggers the effects mentioned above on Y, r and p, which rise from 100.0 to 103.395, 100.0 to 107.829 and 66.66 to 75.374 respectively. During this rising cyclical phase, however, the real interest rate (r - p) decreases as p rises faster than r. This phenomenon swells the inflation tax and will eventually create a budget surplus, which will have a downward effect on interest payments, private wealth and disposable income. Furthermore, the inflationary impact of π resulting from the increase in p should not be overlooked as it will counteract the increase in r, which slows investment.

The repercussions of the drop in real interest rates are therefore a symptom that the fiscal policy's impact was too inflationary, and this plunges the economy into a recession immediately afterwards, as illustrated in Chart 1.

During the recessionary phase, Y, r, p, π, b and W decrease, but the real interest rate rises because p decreases faster than r, slowing the slide of $\dot{b}$ and $\dot{W}$ and possibly even causing them to rise. It is clear from (18) that a new phenomenon will create a deficit, because the position of r relative to p means interest payments and subsidization expenditures will use up any gains from the inflation tax. Disposable income will then rise along with income and the expected rate of inflation. In this new growth phase, p may rise faster than r, lowering the real rate of interest, encouraging a budget surplus and eventually leading to a new economic recession.

This turn of events confirms the observations in the theoretical section on the system's dynamics under a nominally accomodating monetary policy:

1) interest payments and subsidy expenditures constitute a destabilizing element in the model because they vary procyclically (i.e. they rise when the budget registers a deficit, and vice-versa);

2) the inflation tax and tax revenues in general tend to counter-balance the imbalances triggered by servicing the public debt and the subsidy expenses because p is more elastic to the cycle than r.

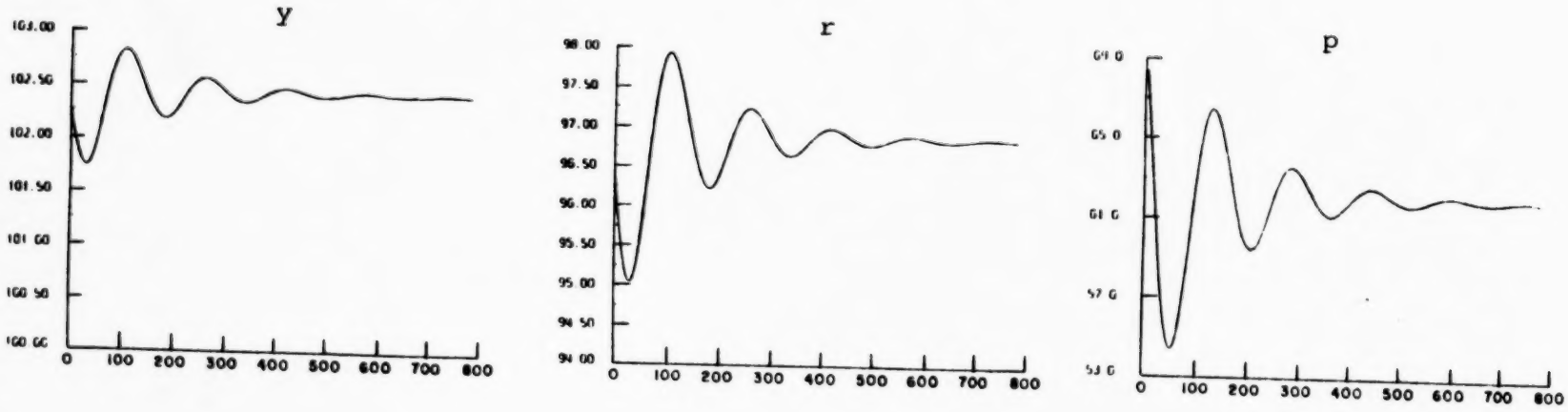
This confirms that a positive wealth effect in the LM curve is destabilizing. If wealth increases under an expansionary fiscal policy, the interest rate will have to increase to maintain equilibrium in the money market, and this will create additional difficulties for p to offset this increase in r and therefore stabilize the system.

The same adjustment dynamics occurs under an expansionary monetary policy (Chart 2). Initially, Y increases from 100.0 to 102.263 and p increases from 66.66 to 68.4530, while r decreases from 100.0 to 96.4432. While b is decreasing following the shock, a very short-term decrease in (r - p) is also observed. It is clear from (18) that b will continue to decrease because of the term (r - p)b.

By studying the structure of equation (18), we see that b again will definitely decrease, and may eventually drag down Y, r and p, leading to a recession. The usual scenario is repeated once more: since p decreases faster than r, the real rate of interest will climb, increasing wealth and eventually stimulating the economy.

Chart 2

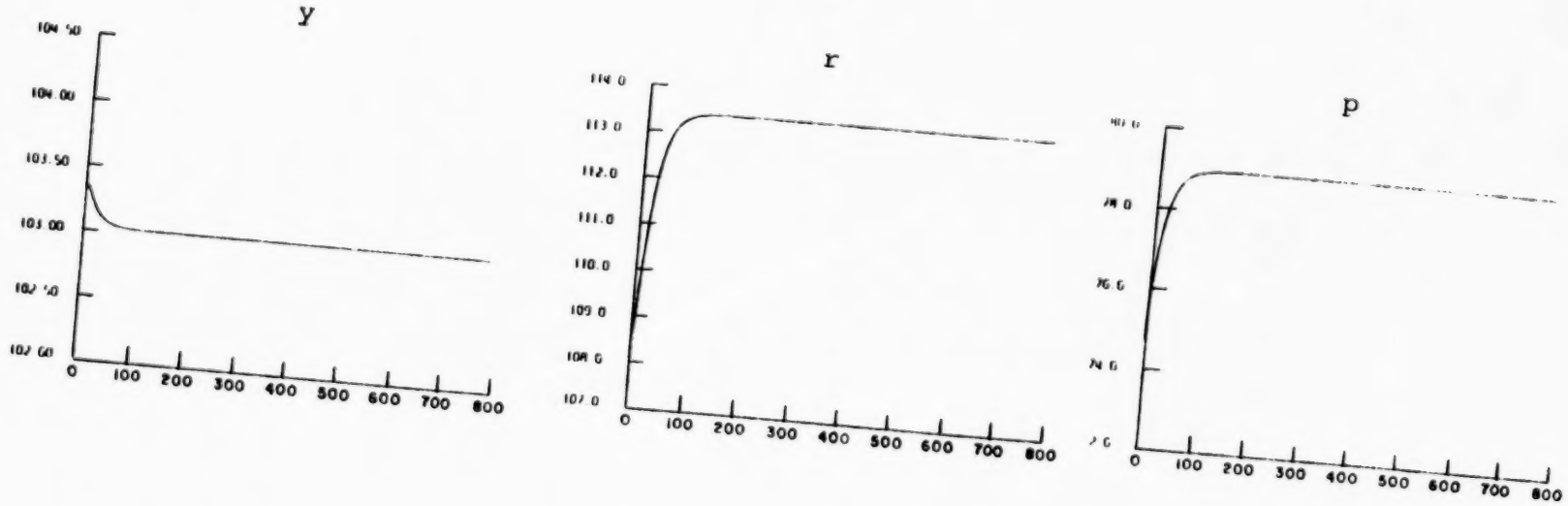
Dynamic Adjustment of an Expansionary Monetary Policy Under a $m=\bar{m}$ Policy, (Scenario 19)[1]



1 C.f. note 30, $y_0=100.0$, $r_0=100.0$ and $p_0=66.66$.

Chart 3

Dynamic Adjustment of an Expansionary Fiscal Policy Under a $b=\bar{b}$ Policy, (Scenario 19)[1]



1 C.f. note 30, $y_0=100.0$, $r_0=100.0$ and $p_0=66.66$.

If these policies are combined with another type of government financial policy, the results contrast sharply with those obtained above. The expansionist fiscal policy is illustrated in Chart 3. Initially, Y increases to 103.295, while r and p increase to 108.619 and 74.7567 respectively, and (r - p) increases slightly. Equation (15a) can be rewritten:

$$(19) \qquad \dot{W} = \dot{m} = G - uY + (r-p)\bar{b} - pm + ur\bar{b} + r\phi[\theta_0 + \theta_1(r-p)(1-\phi)]$$

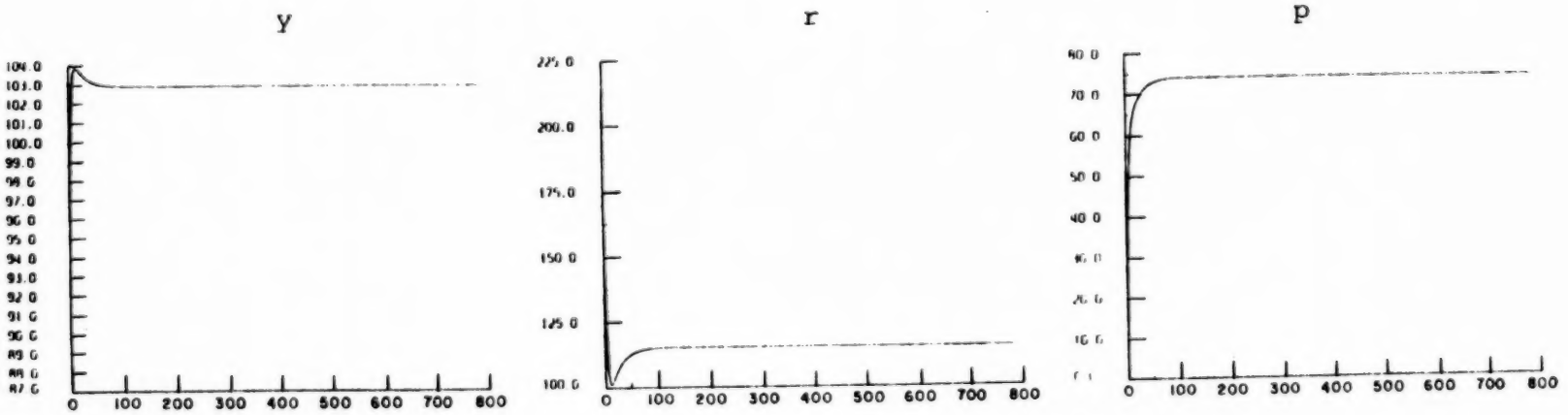
Over the early periods, (r - p) decreases because p increases faster than r and the inflation tax creates budget surplus. At this point, $\dot{W}$ will tend toward smaller values, and may even become negative. Also, since m acts through aggregate demand and the money market, its decrease will halt the growth of Y and p.

This drop in m will prevent r from decreasing as fast as p, and (r - p) will increase. Under a $b=\bar{b}$ policy, this will create a deficit that will stimulate $\dot{W}$ and therefore the entire system.

Chart 4 illustrates the case of an open-market restrictive policy. This produces an immediate drop in Y (88.447) and p (-17.377) and an increase in the nominal interest rate (212.798). This restrictive and deflationary impact is offset by a positive and rising real rate of interest which implies a budget deficit.

Chart 4

Dynamic Adjustment of a Restrictive Monetary Policy Under a $b=\bar{b}$ Policy, (Scenario 19)[1]



1 C.f. note 30, $y_0=100.0$, $r_0=100.0$ and $p_0=66.66$.

This deficit will be financed by an increase in $\dot{W}$ that will stimulate y and p. Then, a budget surplus will eventually be created with the decrease of the real interest rate caused by the reflation. As a result $\dot{m}$ will decrease and so does y.

Before proceeding to the next section, a few remarks are necessary on the system's adjustment dynamics:

1) the $b=\bar{b}$ policy is without a doubt more stable than the accommodating monetary policy; our results demonstrate that the oscillations are still significant in the adjustment process for $m=\bar{m}$ policy;

2) the crucial role played by (r - p) in the system's adjustment dynamics is obvious, confirming the fact that a strong positive ρ has a destabilizing effect.

c) <u>The Interdependence Between Fiscal Policy and Subsidization</u>

We shall now study the impact of subsidization on the effectiveness of fiscal policy. Before moving to the actual analysis, we should review the various mechanisms triggered by an increase in government spending and a rise in the rate of subsidization.

i) Government Expenditures

An increase in government spending generally has a positive effect on the economy. The additional income generated by this spending results in greater demand for money by individuals, placing upward pressure on interest rate levels. The financing of this spending also adds pressure through the competition between borrowers in the financial markets. The rise in interest rates will slow the growth of expenditures and investment, and will diminish the expansionary impact of government spending.

ii) Subsidization

Subsidization is interpreted by producers as a decrease in the cost of borrowed money, or interest rates. This drop in the level of interest actually paid stimulates investment by the subsidized sectors because the number of viable projects increases as the cost of money drops. Once again, financing is required for this expenditure, and this diminishes its positive effect on output.

One distinction must be made for government expenditures, however. The spending on subsidization affects only the recipient sector, while the effects of the financing -- higher interest rates -- affect the entire economy.

It is therefore obvious that each government intervention has a different impact on the economy. What is not clear is how they

are interrelated. Does subsidization influence the fiscal multiplier, or in other words, the effect of government spending on the economy? Does it reinforce the impact of this spending, or weaken it? These are the questions for which we will seek answers. The results of the model simulations will therefore reveal whether the selective subsidization policy detracts from or consolidates the short- and long-term stabilization policy. Assigning different values to the model's variables and parameters results in different scenarios such as those presented in Table 3. These differ in some criteria, which now become important in analysing the interdependence between subsidization and fiscal policy.

a) Rate of Subsidization

Located on a continuum ranging from 0 to 1.0, this percentage indicates the share of interest costs on borrowed money assumed by government. For a real interest rate of 10 per cent, for instance, an investor subsidized at 10 per cent will pay an effective real interest of only 9 per cent, while one subsidized at 90 per cent would pay an effective rate of only 1 per cent.

b) Reactions of Investment to Fluctuations in the Real Rate of Interest

An increase in the real rate of interest discourages investment. In stressing the sensitivity of subsidized and unsubsidized investment, the implications in terms of economic impact are dependent on whether these reactions are significant or not.

c) Reaction of the Demand for Money in Relation to Fluctuations in Interest Rates

If individuals' demand for money is very sensitive to fluctuations in interest rates, it can be shown that fiscal policy will be quite powerful. If the government wishes to finance its expenditures, it need only introduce a small increase in the rate paid on its bonds to be sure of finding takers. It can therefore proceed with spending plans without the necessity of a major increase in interest rates, thus affecting investment only marginally. In a more realistic context, it can be argued that government must significantly increase the return on its bonds to compensate for the transaction costs paid by individuals to transform their assets. The resulting increase in interest rates will therefore discourage investment.[31] The various scenarios selected to illustrate our analysis will therefore centre around these parameters.

Charts 5 and 6 illustrate the way in which the government expenditures multiplier reacts to the rate of subsidization. In scenario (1), the short-term multiplier is about 1.30 and is only partly affected (upwards) by a growing rate of subsidization. This would be termed "weak multiplier elasticity to the rate of subsidization": regardless of the portion of the interest rate paid by government, there will be no substantial change in the impact of its own spending over the fairly short term. We can therefore conclude that while subsidization policy does not adversely affect economic stabilization, neither does it

Chart 5

Short- and Long-Run Relations Between the Rate of Subsidy and Fiscal Multiplier: Normal Cases

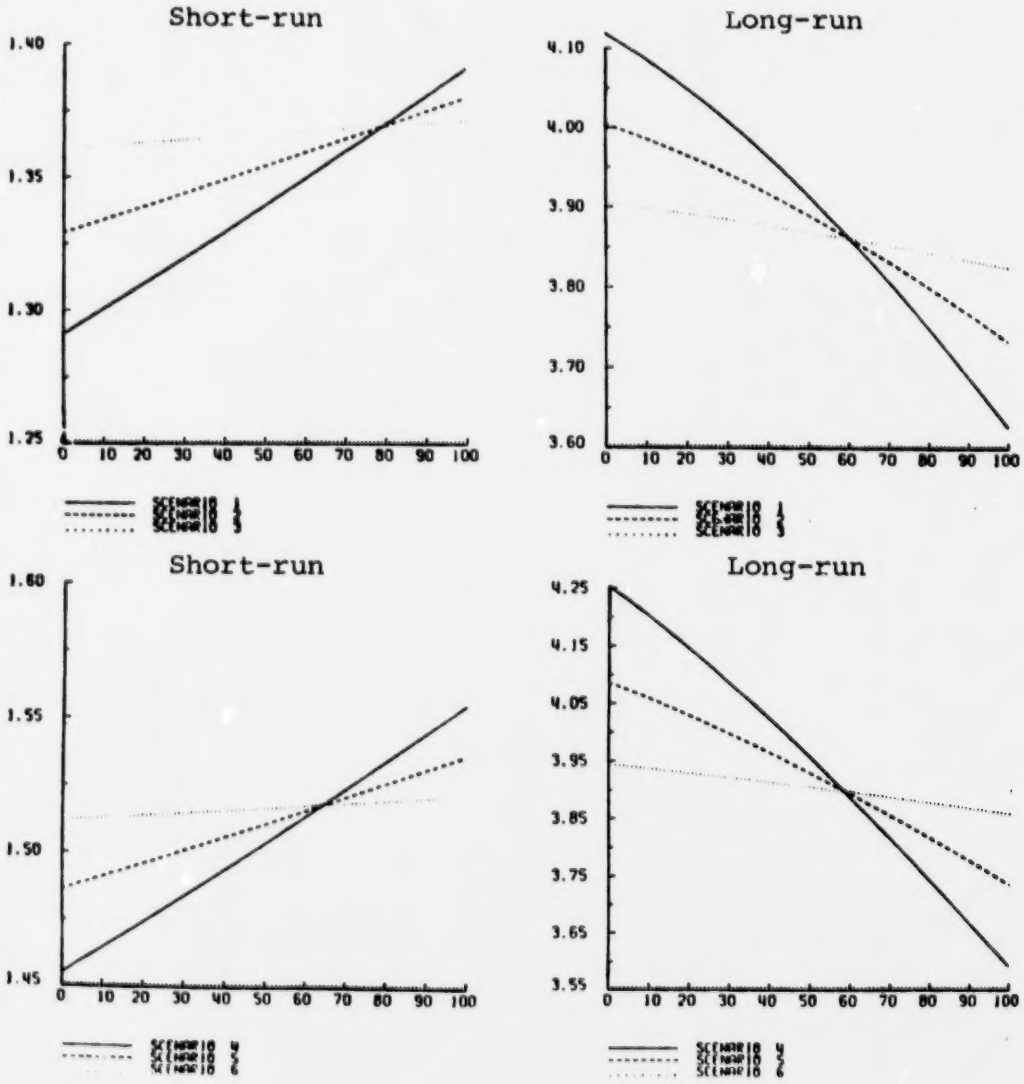
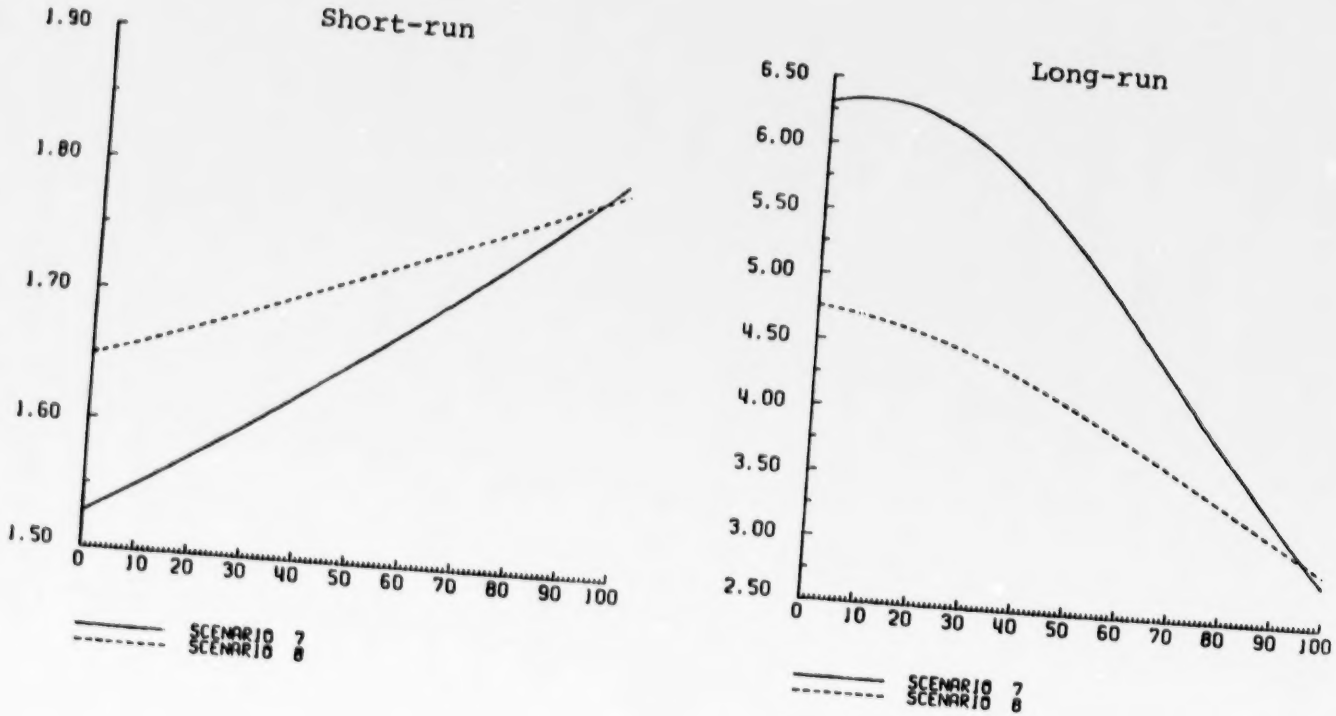


Chart 6

Short- and Long-run Relations Between the Rate of Subsidy and Fiscal Multiplier: Dramatic Cases



constitute an effective instrument to compensate for the ineffectiveness of stabilization policy.

The scenarios are arranged in order of elasticity to subsidized and unsubsidized investment. Scenarios (1, 2, 3) assume that the unsubsidized sectors show very little reaction to fluctuations in the real interest rate, while scenarios (4, 5, 6) assume a strong sensitivity. These two groups are subdivided into three scenarios, in which the subsidized sector's insensitive to variations in the real subsidized interest rate is great (scenarios 1 and 4), average (2 and 5) or small (3 and 6). Scenarios (1) represents the situation in which the unsubsidized sectors are fairly insensitive to changes in the real interest rate, while the subsidized sectors register a high elasticity to the real subsidized interest rate.

The following conclusions apply to all of the scenarios presented for the short term:[32]

1) the value of the multiplier ranges between 1.3 and 1.6;
2) an increase in the rate of subsidization slightly increases the impact of the fiscal multiplier, but not to a significant degree;
3) when subsidized and unsubsidized investors' reaction to interest rate fluctuations is decreasing:
   a) the impact of government spending on the economy increases;

b) the elasticity of government spending to subsidization diminishes;

4) The stronger the reaction of the demand for money is to variations in interest rates, the larger is the fiscal multiplier.

The economic explanation of these observations can be stated in these terms. In the short term, the effects related to financing the deficit have little effect, as the expansionary influences of government spending dominate. The pressures on interest rates resulting from this fiscal measure will have repercussions at a later time. The government therefore finances its public expenditures (including subsidization) partly through an immediate increase in the interest rate and partly through delayed steps to complete the adjustment. The resulting higher interest rate will discourage private investment to a degree equivalent to the investors' sensitivity to variations in the interest rate. Table 1 reveals that the multiplier in scenarios (3) and (6), describing the situation in which subsidized investment is less elastic to interest rates, is generally larger.

It is also clear that as the subsidized investor reacts more to variations in the subsidized rate of interest (scenarios 1 and 4), the subsidization will have a greater effect on the fiscal multiplier. As the investor sees the effective interest rate decline, he will be tempted to proceed with his project <u>a fortiori</u> if his interest rate elasticity is high. The values computed,

however, show that the elasticity of the government expenditures multiplier to subsidization is fairly weak.

Long-Term Analysis

The long-term results contrast somewhat with those for the short-term. First, the inherent effects of the government's budget deficit financing come into full play. The increase in interest rates resulting from the increase in government spending has two effects:

1) the multiplier oscillates between 3.5 and 4.1;

2) the subsidization has a greater effect on the multiplier than in the short-term, but his effect is still small.

The observations for the short-term analysis also apply here. The multiplier is larger because the linkage effects are greater. The multiplier also increases as subsidized investment becomes decreasingly sensitive to the rate of interest, partly isolating itself from interest rate fluctuations.

The elasticity of the fiscal multiplier to subsidization is negative and larger over the long-term, signifying that although it has little effect on fiscal policy, what effect it does have is downward. Completion of the financing cycle gives the results a broader perspective. The larger the rate of subsidization, the

larger the amount of subsidy for a given interest rate, and the greater the needs in financing. This will add to the pressures already exerted on interest rate levels by the financing of expenditures.

Thus, as investment becomes increasingly dependent on variations in the interest rate, the net impact of government spending will diminish. In scenario (4), for instance, subsidization significantly affects the government expenditures multiplier.[33]

In cases where the demand for money is fairly inelastic and where investment (either subsidized or unsubsidized) is highly sensitive to fluctuations in interest rates (scenarios 7 and 8), we found that the subsidization significantly decreased the fiscal policy's long-term impact.[34]

We can therefore conclude that except under strict assumptions for the values of the decisive parameters, subsidization does not significantly affect the government expenditures multiplier in either the short- or long-term.

We can add, however, that with high interest rates, the elasticity of the demand for money to these interest rates might diminish and that the interest elasticity of investment might increase. If, despite this situation, government persists in providing subsidies, the impact of the fiscal policy will probably be weakened. Thus, if government introduces a restrictive fiscal

policy when interest rates are high, it may have to intervene more decisively to counteract the compensation effect of the rate of subsidization on the fiscal policy, and thus achieve its goals.

Conclusion

This paper has attempted to show which are the existing links between a selective subsidization policy and the effectiveness of the fiscal multiplier. After studying the dynamics of fiscal and monetary policies in respect to the form of financing, we proceed to quantify the subsidy-fiscal multiplier relationship and especially the elasticity between the two instruments. Under realistic hypotheses, we found that the instrument which represents subsidization has almost no effect on the fiscal multiplier in either the short or the long run.

However, we discovered that when elasticities of the demand for money and of both subsidized and non-subsidized investment functions decrease and increase respectively, then the subsidy rate reduces the effectiveness of the fiscal multiplier. This important result shows the potential destabilizing effect which subsidization can have even in the presence of elasticity values that would not be considered extreme.

Government should therefore exercise caution when contemplating each individual intervention involving subsidization, and should pay closer attention to the objectives, form and repercussions of its action.

Notes

1 A general study of the macroeconomic impacts of grants of government financial credits (with no distinction from subsidization) is found in M.K. Plantes and D. Small, Macroeconomic Consequences of Federal Credit Activity, Proceedings of the Conference on the Economics of Federal Credit Activity, Part II, April 1980, Washington, D.C.

2 Throughout the analysis, we assume that the real interest rate is positive. However, if necessary, the argument could very well accommodate negative rates.

3 See Ott and Ott (1965) for the endogenization of u into an IS-LM model with a budget constraint. According to Christ (1968); this endogenization can be a determinant condition of the system's stability. However, as Turnovsky (1979) demonstrated, this characteristic is taken into account in equivalent fashion by the inflation tax on wealth.

4 This type of monetary policy is called "passive" or "neutral" in economic literature. See Foley and Sidrauski (1971) and Olivera (1971), among others.

5 For alternative definitions, see Tobin (1970), Sargent and Wallace (1973) and Sargent (1977).

6 Money supply is determined by the central bank and is defined as MS = C + D where C and D represent respectively the cash outside banks and demand deposits. This definition is valid as a preliminary approximation but, in fact, the total money supply is at best an indirect policy instrument. The more direct instruments are: 1) the monetary base; 2) the discount rate; 3) the legal reserve coefficient. While the last two instruments are oriented to the long-term, the monetary base is a short-term instrument operating through the operations of the open market. The monetary base is defined as B = C + R where R represents the reserve amount and therefore

$$\frac{M_s}{B} = \frac{C + D}{C + R} = \frac{C/D + 1}{C/D + R/D} = \tau \text{ is the bank multiplier,}$$

C/D the proportion of money held by households and R/D the proportion of deposits held in reserve. Thus, $M = \tau B$. Note that m is stable, that it is controlled by the central bank and by the banking system and that R/D depends on r. To simplify matters, we assume that $\tau = 1$ (M = B, R = D and R/D = 1). This derivation can be refined by introducing the commercial banks (see Teigen (1964)). This error in specification does not, however, cause any loss in generality.

7 The bar over the letter means that this variable is constant. In this case, the deficit is financed solely through the issue of bonds.

8 For a critical survey of literature on the subject, see Santomero and Seater (1978). For examples of specific applications, see Laidler (1973), Turnovsky (1974), Tobin (1975), Fortin and Newton (1980), Hvidding (1981), Lewis (1981), Nelson (1981), Puichip (1981) and Henry et. al. (1982).

9 We will return to this point later.

10 In fact $\omega = -1/N^* f_{N^*}$.

11 See Lipsey (1978, 1981) and Wilton (1979).

12 From equation (1d), we see that if the monetary policy is accommodating, m is fixed and W varies with b only. If it is bonds that represent the accommodating policy, then we observe the opposite mechanism.

13 If we wish to make the monetary policy "activist" or discretionary, the equation can be specified as follows:

$$\dot{m} = \varepsilon G - uY + (1 - u)\, r\, (W - m) - pW + r\phi\, \theta_0 + \theta_1\, (r - \pi)$$
$$(1 - \phi) + m \quad \text{where}$$

$$\dot{b} = (1 - \varepsilon)\, G - uY + (1 - u)\, r\, (W - m) - pW + r\phi\, \theta_0 + \theta_1$$
$$(r - \pi)(1 - \phi) + m$$

where the last term, m, characterizes an exogenous variation of the monetary base.

14 For example, of macroeconomic models incorporating the dynamics of budget constraint, see Ritter (1955-56), Ott and Ott (1965), Silber (1970), Steindl (1971), Buiter (1976), Tobin and Buiter (1976), Christ (1967-8-9, 1978-9) and Smith (1979).

15 The budget constraint is:

$$\dot{(M/P)} + \dot{(B/P)} = G - T + rB/P + r\phi I_2$$

Equation (12) is derived by using the definitions

$$\dot{(M/P)} = \dot{m} + mp \text{ and } \dot{(B/P)} = \dot{b} + bp \text{ where } \frac{B}{P} = b + \frac{M}{P} = m.$$

16 See Blinder and Solow (1973), Tobin and Buiter (1976), Brunner and Meltzer (1976), Pyle and Turnovsky (1976) and Turnovsky (1977).

17 As Pyle and Turnovsky (1976) pointed out, the best way of including the accumulation of capital in a model such as ours (where the short-term substitutions between labour and capital are ignored) is to define dynamics relative to the capital stock. Use

of this procedure produces the model we have developed except that the budget constraint will be increased by a growth tax. In this case, disregarding the capital stock has few consequences on the results. However, if we wish to introduce capital stock in an adequate manner, we simply have to formulate a more exhaustive model that includes, among other things, a production function, a labour market, capital-labour substitution elasticities, a private sector budget constraint, etc. An example of this type of model in a long-term horizon can be found in Turnovsky (1977), pp. 159-91.

18 If $\partial Y/\partial m > 0$, this means the numerator of $\partial Y/\partial m$ is positive. This numerator is found in $\partial p/\partial m$ premultiplied by the scalar a_1, which implies that $\partial p/\partial m > 0$ if $\partial Y/\partial m > 0$.

19 See Turnovsky (1974) for an exhaustive study of the consequences of an increase in π on the economy.

20 In fact, $\frac{\partial p}{\partial \pi} > 1$ because of the positive effect of π on Y.

21 See Appendix 1.

22 This confirms the conclusions advanced by Turnovsky (1979). See Smith (1981) who, contrary to Blinder and Solow (1973), supports the argument that a small wealth effect in aggregate demand can still be stabilizing.

23 By stability, in this case local, we mean that the characteristic roots have a real negative part, and thus are located in the left-hand side of the complex plan (which includes the real and imaginary roots), and guarantee oscillations of decreasing magnitude. The roots with a nil real part are excluded from the definition of stability because we require an asymptotic type of stability, i.e.,
$\lim t \to \infty + y(t) = 0$
where y(t) is the general solution of a homogenous equation. If, on the other hand, we have a nil real root, this gives rise to an arbitrary constant while a complex root with a nil real part results in an oscillation of constant magnitude which in both cases does not meet the criterion of asymptotic stability. We could have used the Liénard-Chipart stability conditions which greatly simplify the derivation when the degree of the characteristic equation is high. The two methods are equivalent, however. For a thorough analysis of the conditions of stability for differential equations, see Turnbull (1957), Barnett (1973) and Gandolfo (1980). For a macroeconomic application, see Samuelson (1963), pp. 276-83.

24 The effect on r will be greater when α and θ_1 are greater for a given ϕ.

25 $\partial p/\partial W = a_1\{\rho[\gamma_1 + \theta_1(1 - \phi)] + c(1 - u)(W - m) - \beta(c[(1 - u)r - \pi] + \phi)\}/\sigma$

where $\sigma > 0$ is the Jacobian determinant of the system and is equal to its definition in Table 1. The wealth effect of the IS curve is given by $[c[(1 - u)r - \pi] + \phi]/[1 - c(1 - u)] = 0$ and the wealth effect of LM is given by $\rho > 0$. If $[c[(1 - u)r - \pi] + \phi]/[1 - c(1 - u)] = 0$, then $\partial p/\partial W > 0$, which destabilizes the model. On the other hand, if $\rho = 0$, then $\partial p/\partial W > 0$, adding stability to the system.

26 Specifically, the destabilizing elements include interest payments, subsidy expenditures, a wealth effect of positive demand for money and the greater than proportional reaction of p to an increase in π. The stabilizing factors are fiscal receipts, the inflation tax and a positive wealth effect in the IS curve.

27 The data are available from the author on request. The reference scheme is similar for all scenarios in Table 3. Current government expenditures on goods and services, as well as private investment, each account for 20 per cent of Gross National Expenditure; the marginal propensity to consume is 0.6 and the narrow definition of money supply (M1) is used. Finally, the tax rate applied to national income is 0.20.

28 It must be added, however, that for $\rho = 0$, Nguyen and Turnovsky (1979) obtained much higher values than ours for stability (39.52 per cent), while $\rho > 0$, the proportion changed to 0.98 per cent for bond financing.

29 This result is not without precedent. Feldstein (1980) studied the impact of a fiscal policy designed to increase capital intensity in the production process through a non-accelerationist approach for the rate of inflation. Among his various results, the fiscal multiplier became negative over the long-term under some hypotheses.

30 The figures appearing on the ordinate represent Y, r and p as indices (r and p have a common index: 100 = 0.18). Their initial values are 100, 0.18 and 0.12 (66.66 as an index) respectively at t = 0. The figures on the abcissa correspond to the cycle criterion and thus to the unit of time.

31 For fiscal policy to be more effective, the interest elasticity of the demand for money must therefore be as high as possible, while that for investment must be as low as possible.

32 The scenarios were selected on the basis of their uniformity with the basic hypotheses. Different results could, of course, be obtained by changing the values of the various parameters, by two scenarios with different basic hypotheses could not be compared.

33 The value of the elasticity at $\phi = .1$ is -0.66.

34 At the point where $\phi = .9$, the elasticity of the fiscal multiplier to long-term subsidization is -2.7 for scenario 7 and -1.3 for scenario 8. In the latter case, a 10 per cent increase

in the rate of subsidization lessens the impact of government spending by 13 per cent. These scenarios involve more radical assumptions than the others, and this produces different results. For instance, the short run elasticity of the unsubsidized investment function related to real interest rate is -2.68 whereas the one for the subsidized investment function (in respect to the subsidized real interest rates) is -4.0 in scenario 7 and -2.0 in scenario 8. The multipliers for these scenarios may seem high, but they are still valid because the model has no capacity restraint and because we are assuming a continuous flow of idle resources.

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